Interference as an information-theoretic game (Extended abstract)

Sebastian Horvat
Faculty of Physics, University of Vienna
Vienna, Austria
sebastian.horvat@univie.ac.at

Borivoje Dakić
Vienna Center for Quantum Science and Technology (VCQ)
Faculty of Physics, University of Vienna
Vienna, Austria
borivoje.dakic@univie.ac.at

Preprint available at: arXiv:2003.12114

Introduction. The most common way of introducing quantum theory and its basic tenets is via the double-slit experiment, in which a single particle is shown to produce an interference pattern when sent through two slits, in contrast to classical theory in which this effect is missing. Much later, Sorkin [1] analyzed multi-slit experiments (i.e. generalizations of the double slit experiment to three or more slits) and noticed that quantum mechanics does not exhibit higher-order interference, meaning that any measurement pattern produced by a quantum system is reducible to the combination of double-slit interferences. The latter served as a motivation for introducing higher-order interference theories which are defined with respect to the order of interference produced in generalized multi-slit experiments [2]. Such theories are usually formulated within the framework of *generalized probabilistic theories* (GPT-s) [3, 4], with quantum theory being only a particular example. This further motivated the search for a set of intuitive physical principles which could explain why Nature should exhibit at most second-order interference [5, 6, 7, 8].

In this work we reconsider the definitions of multi-slit experiments and Sorkin's classification and notice that: (a) the framework is defined only for single particles (or single systems), (b) the interference order is defined with respect to simple operations of blocking/opening the slits, and (c) the (final) measurement refers exclusively to intensity measurements (average number of particles at a particular location on the screen). Here we generalize this setting to an arbitrary number of particles (systems) and arbitrary set of (local) operations in terms of information-processing tasks. Moreover, the final intensity measurement is replaced by an arbitrary operation/measurement. This generalization offers a shift in the perspective: higher order interference theories are not defined by which phenomena are allowed (e.g. by the structure of interference patterns), but by which tasks can or cannot be accomplished within the theory (i.e. their information-processing capacity). The historical importance of the standard double slit experiment is that it provides a clear demarcation between the classical and the quantum, while multi-slit experiments demarcate quantum and higher-order theories. The aim of our generalizations is to show that this particular class of experiments belongs to a broader class of processes which are more apt for an information-theoretic analysis and which also provide clear cuts between classical, quantum and higher-order theories. Moreover, the latter formulation naturally encompasses processes with multiple systems, thereby enabling the analysis of the relation between interference phenomena and the law of composition.

Information-theoretic reformulation of interference experiments. In order to generalize the standard double-slit experiment to a process defined as an information-theoretic task, let us consider a scenario involving two parties, Alice and Bob, located at opposite sides of a plate pierced by two parallel slits. Alice possesses a single particle that she can send towards Bob and has control of the slits, i.e. she can

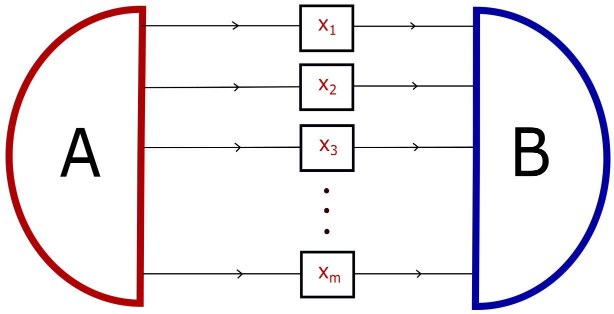

Figure 1: Alice sends her particles/systems towards Bob through m black boxes, which implement local operations depending on Alice's inputs $\{x_1,...,x_m\}$. Upon receiving the systems, Bob performs an arbitrary operation and generates an output bit b.

decide whether to block them or not. On the other side, Bob receives (or not) the particle, performs an arbitrary operation and outputs a bit $b \in \{0,1\}$. We introduce the redefined second order interference term as follows

$$\tilde{I}_2 = \frac{1}{4}\left[P(0|11) - P(0|10) - P(0|01) + P(0|00)\right] = \frac{1}{4}\sum_{x_1,x_2=0}^{1} P(b = x_1 \oplus x_2 | x_1 x_2) - \frac{1}{2}, \quad (1)$$

where $P(b|x_1x_2)$ is the probability that Bob outputs b when the two slits are in states x_1 and x_2 ($x_i = 0,1$ correspond to the i-th slit being respectively blocked or open). The definition $\tilde{I}_2$ reduces to the standard second order interference term (see for instance [1]) if Bob is constrained to output the intensity of the particle inflicted on a point of a screen. On the other hand, the redefined interference term $\tilde{I}_2$ measures the probability of successfully accomplishing the following task: in each run, Alice prepares the slits in state $\vec{x} \equiv \{x_1,x_2\}$ and sends her particle through the slits towards Bob, who, in order to win the game, must output the parity $x_1 \oplus x_2$.

Notice that (1) is formulated in a device-independent way [9], relating only inputs and outputs, without any mention of their underlying physical realization. It is therefore natural to generalize the scenario by replacing slit operations with generic black boxes, which implement arbitrary local operations depending on their inputs. Moreover, instead of constraining Alice to send only single particles, we can generalize her resources to an arbitrary number of systems and analyze how the winning probability depends on the number of systems used in the process. Additionally, the particles/systems sent by Alice can have any internal structure (e.g. spin) that can be accessed by the black boxes.

Analogously to the generalization of the standard double-slit experiment to multi-slit experiments, we can generalize the scenario to an arbitrary number of boxes m. In this case, Alice sends her resources towards Bob, who must output the overall parity of the boxes' inputs, as pictured in Figure 1. Juxtaposed to the standard definition of m-th order interference theories involving the structure of interference patterns produced by single particles in m-slit experiments, our information-theoretic reformulation provides a definition in terms of the probability of winning the parity game involving m boxes. The generalized m-th order interference term is thus

$$\tilde{I}_m = \frac{1}{2^m}\sum_{x_1,\ldots,x_m=0}^{1} P(b = x_1 \oplus \ldots \oplus x_m | x_1 \ldots x_m) - \frac{1}{2}. \quad (2)$$

The reformulated higher order interference theories will thus be characterized by functions $n(k)$, where n refers to the order of interference achievable using k systems. Moreover, in our manuscript we generalize the inputs to be arbitrary *dits*, i.e. elements of a set of cardinality d; however, here we will stick to bits for matters of simplicity.

Classical and quantum systems. If Alice's resources consist of k classical particles, Bob cannot recover the parity of more than k boxes. Therefore, k classical systems can produce at most k-th order interference (in the information-theoretic reformulation).

In our work, we showed that k quantum systems can generate at most $2k$-th order interference, essentially due to the density matrix being a rank-two tensor, thereby linearly coupling the inputs two-by-two. The derivation also relies on the tensor product structure of composite systems; the result could be even more restrictive in the case of indistinguishable systems.

Higher-order theories. We proceeded by first addressing the single-system reformulation of higher-order interference theories within the framework of generalized probabilistic theories, where we formalized the parity game in terms of states, transformations and effects. We then constructed a simple example of how two generic n-th order systems can be used to produce non-vanishing $2n$-th order interference. The latter construction in turn implies that the *lower bound* on interference that can be generated using k systems of orders $(n_1,...,n_k)$ is $\sum_{i=1}^{k} n_i$.

On the other hand, we previously saw that in classical and quantum theory, the *upper bound* on interference of composite systems is also additive, which is tightly related to the tensor product structure of the two theories. This motivated us to inspect generic locally tomographic theories: we proved that the analogous additive relation holds in the case in which the local operations implemented by Alice are single-system operations (i.e. they act independently on the single systems). It remains open whether the same holds in general for multipartite local operations (as it holds e.g. in quantum theory for arbitrary local entangling gates).

References

[1] Sorkin, R.D., 1994. Quantum mechanics as quantum measure theory. Modern Physics Letters A, 9(33), pp.3119-3127.

[2] Dakic, B., Paterek, T. and Brukner, C., 2014. Density cubes and higher-order interference theories. New Journal of Physics, 16(2), p.023028.

[3] Barnum, H., Mueller, M.P. and Ududec, C., 2014. Higher-order interference and single-system postulates characterizing quantum theory. New Journal of Physics, 16(12), p.123029.

[4] Lee, C.M. and Selby, J.H., 2017. Higher-order interference in extensions of quantum theory. Foundations of Physics, 47(1), pp.89-112.

[5] Barnum, H., Lee, C.M., Scandolo, C.M. and Selby, J.H., 2017. Ruling out higher-order interference from purity principles. Entropy, 19(6), p.253.

[6] Ududec, C., Barnum, H. and Emerson, J., 2011. Three slit experiments and the structure of quantum theory. Foundations of Physics, 41(3), pp.396-405.

[7] Niestegge, G., 2013. Three-slit experiments and quantum nonlocality. Foundations of Physics, 43(6), pp.805-812.

[8] Lee, C.M. and Selby, J.H., 2016. Generalised phase kick-back: the structure of computational algorithms from physical principles. New Journal of Physics, 18(3), p.033023.

[9] Brunner, N., Cavalcanti, D., Pironio, S., Scarani, V. and Wehner, S., 2014. Bell nonlocality. Reviews of Modern Physics, 86(2), p.419.