Bipartite post-quantum steering in generalised scenarios

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The discovery of post-quantum nonlocality – i.e., the existence of nonlocal correlations stronger than any quantum correlations but nevertheless consistent with the no-signalling principle – has deepened our understanding of the foundations of quantum theory. The notion of post-quantum non-classicality has recently been extended to the so-called Steering scenarios. Unfortunately, traditional bipartite steering scenarios can always be explained by quantum theory. Here we show that, by relaxing this traditional setup, bipartite steering incompatible with quantum theory is possible. In this QPL talk I will focus on the generalised scenario where Bob also has an input and operates on his subsystem: I will show how in this setup bipartite post-quantum steering is a genuinely new type of post-quantum non-classicality, which does not follow from post-quantum Bell nonlocality.

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Einstein-Podolsky-Rosen steering is a striking nonlocal feature of quantum theory [1, 2]. First discussed by Schrödinger [1], it refers to the phenomenon where Alice, by performing measurements on half of a shared system, remotely 'steers' the state of a distant Bob, in a way which has no classical explanation. From a modern quantum information perspective [2] steering certifies entanglement in situations where Alice's devices are uncharacterised or untrusted, allowing for "one-sided device independent" implementations of information-theoretic tasks, such as quantum key distribution [3], randomness certification [4, 5], measurement incompatibility certification [6, 7, 8], and self-testing [9, 10].

The usefulness of quantum steering as a resource for information processing motivates its comprehensive study, in a similar way to that of other non-classical phenomena, such as Bell nonlocality [11] and contextuality [12]. Here we pursue this operationally from the perspective of a more general theory – which may supersede quantum theory – and explore how to properly understand steering from this more general perspective.

Abstractly, we may view the steering scenario (see Fig. 1(a)) as one where:

- Alice has a device that accepts a classical input, x, that labels the choice of measurement, and produces a classical outcome, a, as the measurement result,
- Bob has a device without an input, that produces a quantum system – the steered system–, which is correlated with the input and outcome of Alice.

Here we are interested in the question of whether a more general theory may allow for steering beyond what quantum theory predicts. That is, could it be possible to find a pair of devices for Alice and Bob which could not be produced within quantum theory, by Alice and Bob sharing a quantum state, upon which Alice performs measurements labelled by x and with outcomes a? The only requirement that we

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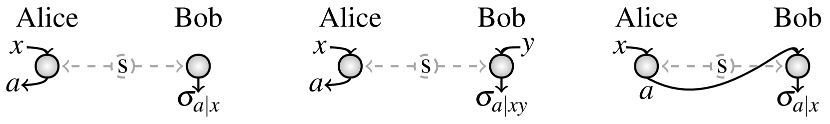

(a) Traditional (b) Bob with input (c) Instrumental

Figure 1: Different generalised bipartite steering setups: (a) The traditional scenario: Alice makes a measurement, steering the state of Bob; (b) The Bob-with-input (BWI) scenario: Bob now also has an input, allowing him to also influence his state, by performing some operation on it; (c) The instrumental steering scenario: BWI where Bob's input now depends on Alice's outcome. Scenario (a) does not admit post-quantum steering, and here we show that scenarios (b) and (c) do.

maintain in this generalised setting is that of relativistic causality: Alice should not be able to use steering to signal to Bob, i.e., to send information to him instantaneously.

A celebrated theorem by Gisin [13] and Hughston, Josza and Wootters [14] (GHJW) shows that post-quantum steering cannot occur in the traditional bipartite setting: any pair of non-signalling devices may always be constructed with quantum components. Despite this, in Ref. [15], post-quantum *multi-partite* steering was discovered: in a tri-partite scenario, Alice and Bob are able to jointly steer the state of a third party, Charlie, in a way which cannot arise from measurements an any quantum state. Subsequently, unified frameworks for studying quantum and post-quantum steering in the multipartite setting have been developed, providing a playground for exploring this fascinating effect [16, 17].

In this work we answer in the positive the outstanding question of whether it is possible to have post-quantum steering in a suitable generalised bipartite scenario. We discover two natural bipartite generalisations of steering that allow for post-quantum effects (see Fig. 1), and in this QPL talk I will discuss the case of Fig. 1(b): here Bob also has an input that allows him to additionally influence his quantum state. I will also show that the post-quantum steering uncovered genuinely constitutes a new effect, that is distinct from post-quantum Bell nonlocality.

Steering preliminaries

In the traditional bipartite quantum steering scenario (see Fig. 1(a)) Alice and Bob share a system in a possibly entangled quantum state ρ. Alice is allowed to perform generalised measurements on her share of the system, which correspond to positive-operator valued measures (POVM). Alice chooses one such measurement $\{M_{a|x}\}_a$, labelled by x, from a set of measurements, and obtains an outcome a with probability $p(a|x) = \mathrm{tr}\left\{(M_{a|x} \otimes \mathbb{I}_B)\rho\right\}$. After the measurement, Bob's unnormalised steered state is $\sigma_{a|x} = \mathrm{tr}_A \left\{(M_{a|x} \otimes \mathbb{I}_B)\rho\right\}$. The collection $\{\sigma_{a|x}\}_{a,x}$ of unnormalised states Bob is steered into is called an *assemblage*. Such assemblages satisfy a no-signalling condition from Alice to Bob, since $\sum_a \sigma_{a|x} = \mathrm{tr}_A\{\rho\} = \rho_B$ independent of x, hence Bob has no information about the choice of measurement Alice made.

Bipartite steering when Bob has an input (Bob-with-Input, Fig. 1(b))

In this generalised scenario, Bob's device also accepts an input (y) before producing a quantum state, as depicted in Fig. 1 (c). One may intuitively think that Bob's system undergoes some transformation before being output from his device. Here, the members of the assemblage will be $\{\sigma_{a|xy}\}_{a,x,y}$.

In the context of quantum theory, the assemblages that can be generated correspond to:

Definition 1. ***Quantum Bob-with-input (BWI) assemblages.***
An assemblage $\{\sigma_{a|xy}\}_{a,x,y}$ *has a has a quantum realisation in the Bob-with-input steering scenario iff there exists a Hilbert space* $\mathscr{H}_A$ *and POVMs* $\{M_{a|x}\}_{a,x}$ *for Alice, a state* ρ *in* $\mathscr{H}_A \otimes \mathscr{H}_B$*, and a collection of CPTP maps* $\{\mathscr{E}_y\}_y$ *in* $\mathscr{H}_B$ *for Bob, such that* $\sigma_{a|xy} = \mathscr{E}_y \left[\mathrm{tr}_A \left\{ (M_{a|x} \otimes \mathbb{I}) \rho \right\} \right]$.

To go beyond quantum theory, we have to identify the natural no-signalling constraints that apply: we must ensure no-signalling from Alice to Bob, and, since Bob has an input, we must also ensure no-signalling from Bob to Alice. These constraints are captured by the following definition:

Definition 2. ***Non-signalling BWI assemblages.***
An assemblage $\{\sigma_{a|xy}\}_{a,x,y}$ *is non-signalling in the Bob-with-input steering scenario iff* $\sigma_{a|xy} \geq 0$ *for all* a,x,y*, and* $\sum_a \sigma_{a|xy} = \sum_a \sigma_{a|x'y} \quad \forall x,x',y$, $\quad \mathrm{tr}\left\{\sigma_{a|xy}\right\} = p(a|x) \quad \forall a,x,y$, $\quad \mathrm{tr} \sum_a \sigma_{a|xy} = 1 \quad \forall x,y$, *where* $p(a|x)$ *is the probability that Alice obtains outcome a when performing measurement x.*

Result 1: Post-quantum steering in the BWI scenario exists. We show that in the case where Alice has ternary inputs ($x \in \{0,2\}$) and binary outputs ($a \in \{0,1\}$), Bob has a binary input ($y \in \{0,1\}$), and the dimension of Bob's Hilbert space is 2, the following assemblage is not quantum:

$$\sigma_{a|xy} = \frac{1}{4}(\mathbb{I} + (-1)^{a+\delta_{x,2}\delta_{y,1}} \sigma_x), \quad \text{where } (\sigma_1, \sigma_2, \sigma_3) = (X, Y, Z) \text{ are the Pauli operators.} \tag{1}$$

Result 2: Post-quantum steering in the BWI scenario, independent from post-quantum Bell nonlocality, exists. We also show that the assemblage Eq. (1) has the following property: whenever Bob performs an (arbitrary) measurement $\{N_b\}$ on it, the observed outcome statistics $p(ab|xy) = \mathrm{tr}\left\{N_b\, \sigma_{a|xy}\right\}$ always have a quantum realisation. To show this, we notice that one may mathematically represent this assemblage as Alice performing Pauli measurements on the maximally entangled state, and Bob applying either the identity or transpose map (which crucially is positive but not completely positive) depending on y. Then, following Ref. [16], the assemblage Eq. (1) can only yield quantum correlations.

Final remarks

In this work we showed how post-quantum steering is not a phenomenon restricted to multipartite scenarios. We also show that the phenomenon is genuinely new since it is independent of post-quantum Bell nonlocality.

The example discussed in this short abstract may be used to construct examples of genuine post-quantum steering for the so-called 'instrumental steering scenario', presented in Fig. 1(c). This scenario arises from the so-called 'instrumental causal network', ubiquitous in causal inference [18, 19]. The key in this construction relies on noticing that the instrumental steering scenario may be understood as a post-selection of the BWI scenario, by conditioning Bob's input choice on Alice's outcome.

Going forward, the most interesting question now is to understand the power of post-quantum steering. For instance, are there information-theoretic or physical principles that are violated by the newly-discovered forms of post-quantum steering found here? are there information processing tasks exploiting post-quantum steering as a resource? Our newly introduced Bob-with-input steering scenario may open the door to exploring bipartite information tasks [20].

We hope that his approach to studying quantum theory 'from the outside' will lead to novel insights into the very structure of quantum theory and the possibilities and limitations of quantum theory for information processing. We expect our results and new insights to contribute to this rapidly developing and exciting field.

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