Quantifying Bell nonclassicality across arbitrary resource types

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In space-like separated experiments and other scenarios where multiple parties share a classical common cause but no cause-effect relations, quantum theory allows a variety of nonsignaling resources which are useful for distributed quantum information processing tasks. These include quantum states, nonlocal boxes, steering assemblages, teleportages, channel steering assemblages, and so on. We introduce a unifying framework which subsumes the full range of nonsignaling resources into a common resource theory: that of local operations and shared randomness (LOSR). Crucially, we allow these LOSR operations to locally change the type of a resource, so that players can convert resources of any type into resources of any other type, and in particular into the type of resource which is most useful for a given operational task. I will discuss how this framework can be used to unify and generalize a large number of results from the literature, and also to quantitatively compare the nonclassicality of resources of different types.

This submission draws from Ref. [1] as well as the companion paper Ref. [2].

1 Introduction

A key focus in quantum foundations is the study of nonclassicality. Starting from the Einstein-Podolsky-Rosen paradox, special focus has been given to experiments involving space-like separated subsystems. In the modern language of causality, the key feature of these scenarios is that the subsystems which are being probed share a classical common cause, but do not share any cause-effect channels between them. In such scenarios, quantum theory allows for distributed quantum channels which act as valuable nonclassical resources for accomplishing tasks which would otherwise be impossible.

2 Types of Resources

The most well-studied examples of resources in Bell scenarios are entangled quantum states [3] and boxes producing nonlocal correlations [4]; but there are many other types of useful resources. We develop a resource-theoretic [5] framework which unifies a wide variety of these, including quantum states [3], boxes [4], steering assemblages [6, 7], teleportages [8, 9], distributed measurements [10], measurement-device-independent steering channels [11], channel steering assemblages [12], Bob-with-input steering channels [13], ensemble-preparing channels, and generic no-signaling quantum channels [14].

All of these can be viewed as instances of distributed quantum channels, in which case the distinction between different **types** of resources is given simply by whether the input and output systems associated with those channels are trivial, classical, or quantum systems. We depict the most common 10 bipartite resource types graphically in Fig. 1, where single wires represent classical systems and double wires represent quantum systems.

Submitted to:
QPL 2018

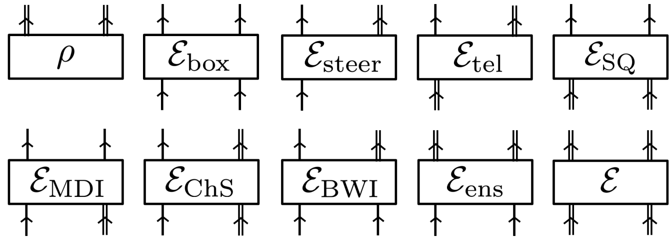

Figure 1: Common types of no-signaling resources, as listed (respectively) in the main text, where classical systems are represented by single wires and quantum systems are represented by double wires.

3 Free resources and free transformations

A resource theory [5] is defined by a set of all processes, together with a set of free processes, where the free processes are all those which are not forbidden by the relevant physical constraints. In our case, we are interested in studying nonclassicality in a common-cause scenario, e.g. a two-party space-like separated experiment, and the relevant constraint is merely that the two parties in question cannot communicate. There are no in-principle constraints on the party's local operations, nor on their ability to share arbitrary (pre-established) common causes. We will take the set of **free processes** (be they resources or transformations) to be those generated by arbitrary local operations and *classical* common causes, so that any object which is *not* of this form requires a *nonclassical common cause*. Hence, we are studying the resource theory of nonclassicality of common-cause processes.

Since 'shared randomness' is a more common term for a 'classical common cause', we see that the free processes in this theory are those that can be generated by local operations and shared randomness (LOSR). It is not hard to see that, for every type of resource, this definition reproduces the standard qualitative distinction between which resources are valuable and which are not. For example, the LOSR-free set of states are the separable quantum states; the LOSR-free set of boxes are the local boxes (that is, those which admit of a hidden variable model), the LOSR-free set of assemblages are the unsteerable assemblages (that is, those which admit of a local hidden-state model), and so on in every case. A selection of free resources is depicted explicitly in Fig. 2.

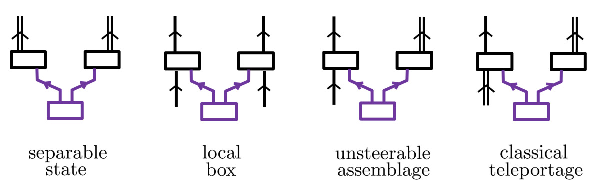

Figure 2: LOSR-free resources are those which can be simulated by local operations (in black) and shared randomness (in purple). We depict four well-studied types of free resources here: separable states, local boxes, unsteerable assemblages, and classical teleportages.

Free transformations, described in more detail in Refs. [1, 2], are those that map resources to resources, *even those of a different type*, and which can be generated by local operations and shared randomness.

4 Quantifying nonclassicality across types

Any resource which *cannot* be generated by LOSR operations is said to be nonfree or valuable, and is necessarily generated by a nonclassical common cause process. Resource theories provide a rigorous means of quantifying the value of all such objects. Namely, a resource is said to be *at least as nonclassical as* another resource if it can be transformed to the second using free transformations. The set of free transformations induces a preorder on the space of all resources, and this is typically the main object of interest in a resource theory. Because the preorder is often quite complicated, one often resorts to monotones or witnesses to extract relevant (but incomplete) information about the preorder.

Our innovation here is to make such quantitative comparisons among *all* resources in Bell scenarios, *regardless of their type*. In keeping with this, we introduce several type-independent monotones, and apply these to begin characterizing aspects of the type-independent preorder. As an illustrative example, in Ref. [1] we fully characterize the relative nonclassicality of five distinct resources of four different types, including an entangled state, a distributed measurement, two steering assemblages, and a box. We also introduce a hierarchy of semidefinite programs (SPDs) which allow one to witness the nonclassicality of any nonfree resource of any type, and we discuss how this hierarchy may also be applied to approximate the value of monotones (which might otherwise be difficult to compute).

5 Transforming among different manifestations of nonclassicality

Having a type-independent framework for studying different manifestations of nonclassical common causes also allows us to consider the possibilities for converting from one manifestation of nonclassicality to another. For instance, one can ask whether every entangled state can be converted into a valuable box-type resource. As illustrated by the existence of Werner states which cannot violate Bell inequalities [15, 16], the answer is no. Similarly, one could ask whether every entangled state can be converted into a valuable teleportage-type resource; equivalently: is every form of entanglement useful for some teleportation experiment? In this case, the answer is yes [9]! Furthermore, every entangled state can be converted to some valuable distributed-measurement-type resource [17]. This last fact has been shown to imply the possibility of measurement-device-independent tests of entanglement [18] and steering [11].

Building on these results, in Ref. [1] we undertake a systematic study of which types of resources can always be converted to some valuable resource of another type, subsuming and generalizing the examples given just above. We also show that each such possible class of conversions implies the possibility of reducing the assumptions required to characterize resources of the relevant type. Leaving the relevant details and definitions to Ref. [1], we can informally state one of our main results here:

Theorem 1. *Every resource* of every type *can be transformed into some distributed measurement which is exactly as nonclassical; that is, such that the two are freely interconvertible under free operations.*

In other words, any nonfree resource of any type can be used to implement some distributed measurement which could not be implemented by noncommunicating parties who share only classical common causes. A key practical consequence of this theorem, as we show in Ref. [1], is that *every* resource can be characterized in a measurement-device-independent fashion.

Acknowledgment—This research was supported by a Discovery grant of the Natural Sciences and Engineering Research Council of Canada and by Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through the Department of Innovation, Science and Economic Development Canada and by the Province of Ontario through the Ministry of Research, Innovation and Science.

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