A mathematical framework for operational fine tunings

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Although quantum mechanics is about a century old, there is still not a universal consensus on what is the nature of the reality it describes. Multiple interpretations are still on the table, each giving a radically different account of the world [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In order to obtain an uncontroversial understanding of how nature behaves according to quantum theory, it is useful to first understand which of its features are inherently nonclassical. A rigorous way to achieve this is to formalize precise notions of classicality and formulate theorems that prove their inconsistency with the theory. This approach is successfully implemented in the framework of ontological models [12], where assuming natural notions of classicality (that in fact hold in classical Hamiltonian mechanics [13]), like local causality [14] and noncontextuality [15, 16], leads to contradictions with the statistics of quantum theory. As a consequence, nonlocality and contextuality emerge as truly nonclassical features. A related one, which has recently been discovered and that we here consider, is the breaking of time symmetry [17]. No-go theorems of these kind do not exist for other phenomena usually listed as characteristic and surprising novelties of quantum theory, like superposition, entanglement, interference and no cloning. It is indeed possible to show that they can be reproduced, still allowing for a natural interpretation, in classical-like theories in the phase space such as Spekkens' toy model [18, 19].

In this work we focus on the aspect that is common among all such inherently nonclassical features of quantum theory and we develop a mathematical framework for it. More precisely, the mystery characterizing these features can be distilled in the fact that they involve fine tuned properties – or, for simplicity, *fine tunings*. These are defined as properties that hold at the operational level, but cannot hold at the ontological level (they only hold by fine tuning of the ontic parameters). Let us consider a paradigmatic example. Bell's theorem can be explained by appealing to the failure of parameter independence, one of the components in the assumption of Bell's local causality [14]. Accepting this explanation would mean that no signaling holds at the level of the operational statistics, but it does not hold at the ontological level, due to the presence of superluminal causation. In this case no signaling would emerge as a fine tuned property, because it would only arise from a fine tuning of the ontic parameters. The intuition that the conflict between the fundamental nonlocality of quantum mechanics and the operational validity of Einstein's relativity is resolved at the price of introducing what we here call a fine tuning was originally highlighted by Valentini [20]. Let us consider a second example. The generalized notion of noncontextuality [16] states that experimental procedures that are, in principle, operationally indistinguishable must be represented by the same probability distribution at the ontological level. Accepting that quantum mechanics is contextual would mean to accept that

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the operational equivalences are a result of a fine tuning of the ontic parameters, that makes two distinct ontological probability distributions look the same at the operational level.

Accepting fine tuned properties is problematic in a scientific theory, because they characterize nature with a conspiratorial connotation, meaning that there are some features that are not available for us to use and experience, even if they are present in nature. This denies the core idea of science and its empiricist roots.

In 2015 Wood and Spekkens [21] introduced the notion of fine tunings in the framework of classical causal models [22] and showed that there are no explanations of Bell's theorem free of fine tunings. The requirement of no causal fine tuning reads as a criterion for the most natural causal explanation of the observed conditional independencies between the variables. In contrast, in this work we refer to fine tunings that are purely operational, without any assumption on the underlying causal structure. In brief, the condition of no operational fine tuning requires the operational equivalences between the statistics of experiments to be preserved at the ontological level. Here the ontological level is defined by the notion of ontic extension [17], which removes the causal assumptions from the standard ontological model framework (often called the hidden variable model framework) [12]. Common examples of operational fine tunings are the ones described above – parameter independence and generalized noncontextuality. Underlying the distinction, so far ignored, between causal and operational fine tunings allows to further dissect the assumptions present in the no go theorems and to obtain insights regarding the relations between nonclassical features, as we will show in the following.

The requirement of no operational fine tuning can be seen as a requirement of structure preservation between the operational and the ontological level. The branch of mathematics that deals with structures preservation is the one of category theory and functors [23], that we here adopt. More precisely, we define the operational and ontological categories. The former refers to all the possible experimental statistics associated to experiments, while the latter refers to the corresponding ontological representations. An operational theory, like quantum mechanics, is associated to a subcategory of the operational category. A property, represented by an equation in such subcategory, is no fine tuned with respect to a functor – that is associated to the ontological representation of the operational theory – if the functor maps it to the analogous equation in the ontological category.

In summary, we provide a rigorous mathematical framework, developed also in the language of category theory and functors, that characterizes operational fine tunings. In addition to accounting for all the known operational fine tunings – generalized noncontextuality, parameter independence and time symmetry – the framework describes more general ones, thus setting the ground for formulations of further no-go theorems. In light of our framework and the distinction we draw between operational and causal fine tunings, we analyze the notion of Bell's local causality, that is composed by an operational fine tuning – parameter independence – and a purely causal fine tuning – outcome independence [24]. In this way we deepen the understanding of the relation between nonlocality and generalized contextuality, where the former is not just an example of the latter – as usually stated if considering the Kochen-Specker notion [15] – because it can be obtained, unlike contextuality, by involving a purely causal fine tuning.

The current work originates a proper research program, where the next step consists of formulating a resource theory for operational fine tunings. This would also allow us to witness and quantify the presence of fine tunings in information processing tasks and quantum computational protocols. As many results are showing [25, 26, 27, 28, 29, 30, 31, 32, 33], the quantum phenomena proven to be responsible for the quantum computational advantages are so far dependent on the model and scenario considered, and this because, by construction, these phenomena arise only in certain setups. In this respect, the benefit of adopting the notion of fine tunings, is that it captures the aspect that is common and inherently nonclassical about all such physical phenomena. Therefore, it may be possible that a certain amount of fine tuning, independent on which actual phenomenon is manifested in the setup considered, is necessary (or even sufficient!) for quantum computational advantages. For these reasons, we believe that this notion is more promising than the ones so far explored in order to understand what powers quantum computers and technologies. Finally, the very foundational motivation for studying and characterizing fine tunings is to ultimately develop a new ontological framework for quantum theory free of fine tunings, or, alternatively, explain them as emergent from physical phenomena. [1]

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[1] An example of emergent fine tuning has been proposed in [17], where the no retrocausality fine tuning can be thought as originating from thermodynamics reasons similar to the ones proposed for explaining the arrow of time. Another famous example is present in Valentini's version of Bohmian mechanics [34].

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