Resource Theories of Communication

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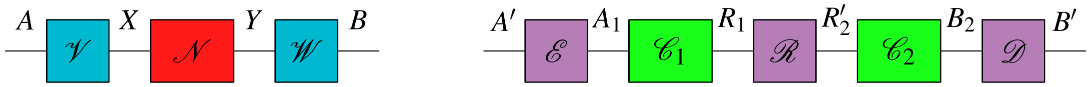
A series of recent works has shown that placing communication channels in a coherent superposition of alternative configurations can boost their ability to transmit information. Instances of this phenomenon are the advantages arising from the use of communication devices in a superposition of alternative causal orders, and those arising from the transmission of information along a superposition of alternative trajectories. The relation among these advantages has been the subject of recent debate, with some authors claiming that the advantages of the superposition of orders could be reproduced, and even surpassed, by other forms of superpositions. To shed light on this debate, we develop a general framework of resource theories of communication. In this framework, the resources are communication devices, and the allowed operations are (a) the placement of communication devices between the communicating parties, and (b) the connection of communication devices with local devices in the parties' laboratories. The allowed operations are required to satisfy the minimal condition that they do not enable communication independently of the devices representing the initial resources. [This is an extended abstract of [14]].

1 Introduction

Quantum Shannon theory describes communication where information is encoded in quantum states. In a series of recent works, a generalisation of quantum Shannon theory has been proposed where not only the information carriers, but also the configuration of the transmission lines can be in a quantum superposition. In particular, communication channels can be combined in a superposition of different causal orders [10, 20, 4, 12, 18, 19, 16, 22], using an operation known as the quantum `SWITCH` [3, 7], or in a similar spirit, information can be sent along a superposition of trajectories [11, 1, 8, 15] leading to superpositions of alternative quantum evolutions [2, 17, 8]. Both of these types of coherent control over channel configurations have been shown to yield a wide range of communication advantages in comparison to standard quantum Shannon theory.

Recently, the works [10, 20, 4] on the superposition of causal orders have been criticised on the grounds that similar advantages could be obtained with coherent control of communication devices [1], or coherent control of encoding and decoding operations [13]. Whilst we note that Refs. [10, 20, 4] in fact only claimed that the superposition of causal orders offers an advantage with respect to *standard* quantum Shannon theory, where communication devices are composed in a definite order and no coherent control over their configuration is allowed, these criticisms motivate a comparison between different types of coherent control: (1) coherent control over the choice of communication channels [11, 1, 8], (2)

Submitted to:
QPL 2020



(a) (b)

Figure 1: (a) An illustration of the basic placement supermap $\mathscr{S}^{A,B}_{\text{place}}(\mathscr{N}) := \mathscr{W}^B \circ \mathscr{N} \circ \mathscr{V}^A$. (b) An illustration of the encoding-repeater-decoding supermap $\mathscr{S}_{\mathscr{E},\mathscr{R},\mathscr{D}}(\mathscr{C}_1 \otimes \mathscr{C}_2) := \mathscr{D} \circ \mathscr{C}_2 \circ \mathscr{R} \circ \mathscr{C}_1 \circ \mathscr{E}$.

coherent control over the causal order of communication channels [10, 20, 4], and (3) coherent control over encoding and decoding operations [13].

In this paper, we construct a general framework for resource theories [9] of communication, which captures the differences between these three types of control, and more generally enables a formal construction of novel communication paradigms.

2 Standard Quantum Shannon Theory as a Resource Theory

The proper way to interpret the communication advantages shown in Refs. [10, 20, 4, 12, 18, 19, 16, 21, 11, 1, 8, 15] is to regard them as comparisons between different resource theories of communication. In a resource theory, the set of all possible resources is described by a set of objects, equipped with a set of operations M acting on them, which are closed under sequential and parallel composition. The basic idea of the resource-theoretic framework is to define a subset of operations $\mathsf{M}_{\text{free}} \subseteq \mathsf{M}$, which are regarded as *free*.

In the context of communication, we take the set of objects to be the set of all quantum channels, and the set of operations M to be the set of all *quantum supermaps* (higher-order transformations that transform quantum channels to quantum channels) [5, 6]. Mathematically, the former are objects in a symmetric monoidal category, and the latter are the corresponding morphisms.

We cast standard quantum Shannon theory in the form of a resource theory [9], by specifying the free supermaps M_{free}. Formally, a communication device is a quantum channel $\mathscr{N} \in \mathsf{Chan}(X,Y)$, transforming systems of type X into systems of type Y, without assigning to them a specific physical location. Accordingly, we will call the systems X and Y unplaced systems, and the channel $\mathscr{N} \in \mathsf{Chan}(X,Y)$ an *unplaced channel*.

The placement of a communication device between a sender and a receiver is described by a *placement supermap*. That is, a quantum supermap from $\mathsf{Chan}(X,Y)$ to $\mathsf{Chan}(A,B)$, where system A (B) is isomorphic to system X (Y), and is placed at the sender's (receiver's) end. Placement supermaps are interpreted as being performed by a communication provider or Nature itself.

In standard quantum Shannon theory, the *basic placement supermap* (Fig. 1a) is defined as:

$$\mathscr{S}^{A,B}_{\text{place}}(\mathscr{N}) := \mathscr{W}^B \circ \mathscr{N} \circ \mathscr{V}^A\,, \tag{1}$$

where $\mathscr{V}^A \in \mathsf{Chan}(A,X)$ and $\mathscr{W}^B \in \mathsf{Chan}(Y,B)$ are channels implementing the isomorphisms $A \simeq X$ and $Y \simeq B$, respectively. We will call the systems A and B *placed systems* and the channel $\mathscr{C} := \mathscr{S}^{A,B}_{\text{place}}(\mathscr{N})$ a *placed channel*.

For communication through k communication devices $\mathscr{N}_i \in \mathsf{Chan}(X_i,Y_i)$ the placement of these devices in parallel, or in sequence via a repeater (with access to placed systems R,R'), is described by a parallel composition of the appropriate basic placement supermaps:

$$\mathscr{S}^{\mathbf{A},\mathbf{B}}_{\text{par}}(\mathscr{N}_1,\ldots,\mathscr{N}_k) := \mathscr{S}^{A_1,B_1}_{\text{place}}(\mathscr{N}_1) \otimes \cdots \otimes \mathscr{S}^{A_k,B_k}_{\text{place}}(\mathscr{N}_k) \tag{2}$$

$$\mathscr{S}^{A,R,R',B}_{\text{seq}}(\mathscr{N}_1,\mathscr{N}_2) := \mathscr{S}^{A,R}_{\text{place}}(\mathscr{N}_1) \otimes \mathscr{S}^{R',B}_{\text{place}}(\mathscr{N}_2)\,, \tag{3}$$

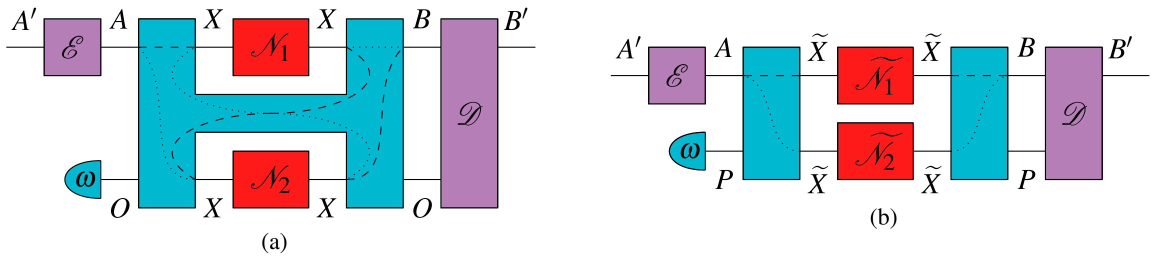


Figure 2: (a) Communication through the quantum `SWITCH`. (b) Communication through the superposition specified by vacuum extensions of two channels.

The difference between parallel and sequential placement is only in the different spacetime points in which the inputs and outputs of the channels are placed. Once the devices have been placed in a given configuration between the communicating parties, the communicating parties can perform local operations on the inputs/outputs accessible to them. These operations are described by a *party supermap*, that is, a supermap on the set of placed channels.

In the example of a sequential placement of two channels (3), without access to auxiliary systems, the communicating parties Alice, Ray, and Bob can together perform the party supermap

$$\mathscr{S}_{\mathscr{E},\mathscr{R},\mathscr{D}}(\mathscr{C}_1 \otimes \mathscr{C}_2) := \mathscr{D} \circ \mathscr{C}_2 \circ \mathscr{R} \circ \mathscr{C}_1 \circ \mathscr{E}\,. \tag{4}$$

which we call *encoding-repeater-decoding* (Fig. 1b). This can be generalised to communication through any `DAG` of $k \geq 2$ devices.

3 General Resource Theories of Communication

The above formulation of standard quantum Shannon theory motivates a general definition:

Definition 1. (Resource theory of communication.) *A resource theory of communication is specified by a set* M_{free}*, closed under sequential and parallel composition, containing (1) supermaps from unplaced channels to unplaced channels, (2) supermaps from unplaced channels to placed channels, and (3) supermaps from placed channels to placed channels.*

Intuitively, given any set of initially available channels, M_{free} describes the valid ways in which these channels can be used. The set M_{free} can be specified by a generating set of operations [9], which represent the basic types of free operations allowed in the theory. For example, standard quantum Shannon theory is constructed from *basic placement* (1) and *encoding-repeater-decoding* (4).

We argue that every meaningful resource theory of communication should satisfy a minimal requirement: the free operations should not allow the sender and receiver to communicate independently of the original communication devices. If this were the case, then it would trivialise the notion of communication enhancement. Formally, we require that any resource theory of communication M_{free} must not contain side-channel generating operations:

Definition 2. (Side-channel generating operations.) *A supermap* $\mathscr{S} \in \mathsf{M}$ generates a classical (quantum) side-channel *if there exist two free supermaps* $\mathscr{S}_1 \in \mathsf{M}_{\text{free}}$ *and* $\mathscr{S}_2 \in \mathsf{M}_{\text{free}}$ *such that, for all choices of input channels* $(\mathscr{N}_1, \ldots, \mathscr{N}_k)$ *for supermap* $\mathscr{S}_1$*, one has*

$$(\mathscr{S}_2 \circ \mathscr{S} \circ \mathscr{S}_1)(\mathscr{N}_1, \ldots, \mathscr{N}_k) = \mathscr{C}\,, \tag{5}$$

where $\mathscr{C}$ *is a placed quantum channel with non-zero classical (quantum) capacity.*

Two examples of resource theories which generalise standard quantum Shannon theory are the ones which include the coherent control over the order of channels, or over the choice of communication channel. In these cases, we define the operations which combine quantum channels (a) in a superposition of orders (Fig. 2a), and (b) in superposition of alternative trajectories (Fig. 2b), as new types of placement supermaps.

Applying this resource-theoretic framework, we argue that (a) the comparison between the superposition of causal orders and superposition of communication channels proposed in Ref. [1] is uneven, because the superposition of communication channels requires stronger initial resources than the superposition of causal orders, and (b) the examples of communication with control over encoding and decoding proposed in Ref. [13] do not satisfy the minimal requirement of a resource theory of communication.

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