Flow Conditions for Continuous Variable Measurement-based Quantum Computation

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Abstract. We introduce flow-based methods for continuous variables graph states, which we dub CV-flow. These are inspired by, but not equivalent to, the notions of flow and g-flow for discrete variables measurement-based quantum computing (MBQC). We show that if an MBQC with CV-flow has the same number of inputs as outputs, then it implements a unitary in a suitable sense. Like for discrete variables, these constructions are useful for determining when an arbitrary CV graph state can be used for a practical computation and investigating the trade-off between classical and quantum depth, which has led to depth complexity separation between MBQC and circuit based quantum computing. In developing our proofs, we also provide a method for converting a CV-MBQC computation into a circuit form, analagous to the circuit extraction techniques of Miyazaki et al. Our results and techniques naturally cover the cases of MBQC for quantum computation with qudits of prime local dimension.

Keywords: MBQC, Continuous Variables, Flow

Causal flow is a graph-theoretical tool for characterising the quantum states used in measurement-based quantum computation (MBQC) and closely related to the measurement calculus [1–4]. Its original purpose was to identify a class of states that can be used to perform a deterministic MBQC despite inherent randomness in the outcomes of measurements, but it has since found applications to a wide variety of problems in quantum information theory.

Along with its generalisation g-flow [5], it has been used to parallelize quantum circuits by translating them to MBQC [6], to construct schemes for the verification of blind quantum computation [7, 8], to extract bounds on the classical simulatability of MBQC [9] and to prove depth complexity separations between the circuit and measurement-based models of computation [6, 10]. A relaxation of these notions was also used in [11] to further classify which graph states can be used for MBQC. g-flow can also be viewed as a method for turning protocols with post-selection on the outcomes of measurements into deterministic protocols without post-selection. This perspective has been used for the verification of measurement-based quantum computations [12], as well as state of the art quantum circuit optimisation techniques [13] and even to design new models of quantum computation [14].

Concurrently, it has become apparent that quantum computing paradigms other than qubit based models might offer viable alternatives for constructing a quantum computer. Continuous variables (CV) quantum computation, which has a physical interpretation as interacting modes of the quantum electromagnetic field, is such a non-standard model for quantum computation that has recently been gaining traction [15, 16]. The MBQC framework has been extended to the CV case, with a surprisingly similar semantics [17, 18]. Accordingly, some structures transfer naturally from DV to CV, and it is of interest to investigate if it is possible to define notions of flow for CV-MBQC. However, CV-MBQC comes with an additional complication: convergence in the limit of infinite squeezing. These limits are implicit in CV teleportation protocols, but the convergence of an MBQC with arbitrary entanglement topologies is not assured.

In this paper, we define such a notion, converting the results on flow and g-flow from [5] to continuous variables. Furthermore, we construct examples to compare our CV flow conditions to the original DV conditions. On the way, we develop a method for circuit extraction, similar to that for DV by Miyazaki et al [10]. Finally, we see that all our results follow naturally for the qudit case when the local dimension is prime.

1 CV-MBQC

1.1 Continuous variables

In CV quantum computation, the basic building block is the *qumode*[1], an infinite-dimensional Hilbert space $\mathrm{L}^2(\mathbb{R})$ of square-integrable functions along with a pair of unbounded linear position and momentum operators Q and P, which are defined on the subspace $\mathcal{S}(\mathbb{R}) \subseteq \mathrm{L}^2(\mathbb{R})$ of Schwartz functions. From these, we can define the corresponding translation operators:

$$\mathsf{X}(s) \coloneqq \exp(-is\mathsf{P}) \quad \text{and} \quad \mathsf{Z}(s) \coloneqq \exp(is\mathsf{Q}), \tag{1}$$

These are further related by the Fourier transform operator $\mathsf{F} : \mathcal{S}(\mathbb{R}) \to \mathcal{S}(\mathbb{R})$:

$$\mathsf{F}\mathsf{Q}\mathsf{F}^\dagger = \mathsf{P} \quad \text{and} \quad \mathsf{F}\mathsf{P}\mathsf{F}^\dagger = -\mathsf{Q}. \tag{2}$$

The quantum state of a set of qumodes can be used to encode information and perform computations just as

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[1] This terminology comes from quantum optics, where we can identify each quantisation mode of the quantum electromagnetic field with a space $\mathrm{L}^2(\mathbb{R})$.

one would with a set of qubits, using operations from the set

$$\{\mathsf{F}_j, \exp(is\mathsf{Q}_j), \exp\big(is\mathsf{Q}_j^2\big), \exp\big(is\mathsf{Q}_j^3\big), \exp(i\mathsf{Q}_j\mathsf{Q}_k)\}, \quad (3)$$

for any $s \in \mathbb{R}$ (the indices j, k indicate on which qumodes the unitary acts). These form a gate set that is universal for the unitary group whose Lie algebra is the set of polynomials in $\{\mathsf{Q}_j, \mathsf{P}_j\}$ with real coefficients. Put otherwise, it is possible to generate the evolution corresponding to any Hamiltonian which can be written as a polynomial in $\{\mathsf{Q}_j, \mathsf{P}_j\}$ [15]. For brevity and by analogy with DV, we write:

$$\mathsf{Z}_j(s) \coloneqq \exp(is\mathsf{Q}_j), \quad (4)$$

$$\mathsf{X}_j(s) \coloneqq \exp(is\mathsf{Q}_j) = \mathsf{F}_k \mathsf{Z}_j(s)\, \mathsf{F}_k^\dagger, \quad (5)$$

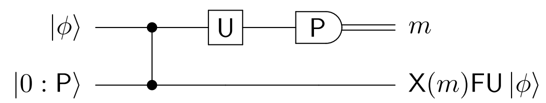
$$\mathsf{CZ}_{j,k}(s) \coloneqq \exp(is\mathsf{Q}_j\mathsf{Q}_k), \quad (6)$$

$$\mathsf{CX}_{j,k}(s) \coloneqq \exp(is\mathsf{Q}_j\mathsf{P}_k) = \mathsf{F}_k\, \mathsf{CZ}_{j,k}(s)\, \mathsf{F}_k^\dagger, \quad (7)$$

$$\mathsf{U}_k(\alpha, \beta, \gamma) \coloneqq \exp(i\alpha\mathsf{Q}_k) \exp\big(i\beta\mathsf{Q}_k^2\big) \exp\big(i\gamma\mathsf{Q}_k^3\big). \quad (8)$$

1.2 Gate teleportation

The workhorse of MBQC (in DV and CV) is gate teleportation, which makes it possible to apply a unitary operation from a specific set on a qumode by entangling it with another qumode and measuring. Using the analogue of the controlled-Z gate we can generate entanglement between two qumodes in CV. Informally, the quantum circuit for teleportation in CV is:



Here, the two qumode interaction is CZ_{12}, U is any unitary gate that commutes with CZ_{12}, and we measure the first qumode in the P basis. The auxiliary state $|0 : \mathsf{P}\rangle$ is a momentum "eigenstate" with eigenvalue 0 and corresponds to a Dirac delta distribution center at 0 in the function representation. If we view U as a change of basis for the measurement, this "gadget" allows us to perform universal computation using only entanglement and measurements, in the sense of Braustein and Lloyd [15, 18]. In particular, it is possible to embed DV quantum computing into this model using the GKP encoding [19–21].

Measurement error There is an extra gate $\mathsf{X}(m)$ on the output of the computation, which depends on the result of the measurement. We dub this the "measurement error", and, loosely speaking, correcting for this error is the goal of this article. More generically, the question is: given a highly entangled state, obtained by applying gates of the type CZ between an arbitrary but finite number of qumodes, is it possible to measure the qubits such that we can always correct for the resulting measurement errors? In such a scheme, the error spreads to more than one adjacent node and correcting it requires a more subtle strategy.

Squeezing Formally, because Dirac deltas are not square-integrable functions, it is necessary to use an approximation to the momentum "eigenstate" $|0 : \mathsf{P}\rangle$. We use the *squeezed state* parametrized by a positive real squeezing parameter η:

$$\mathsf{S}(\eta)\,|0\rangle \coloneqq \exp\Big(i\frac{\eta}{2}(\mathsf{QP} + \mathsf{PQ})\Big)\,|0\rangle \in \mathcal{S}(\mathbb{R}), \quad (9)$$

where $|0\rangle$ is the vacuum state (not to be confused with the zero vector). In the momentum representation , this is a Gaussian distribution in the momentum variable p, centered at the origin $p = 0$ and with width $e^{-2\eta}$. It is well known that in the limit $\eta \to \infty$, these states give a representation of Dirac deltas.

1.3 Graph states

A (CV) **open graph** is an undirected $\mathbb{R}$-edge-weighted graph[2] G, along with two subsets of nodes I and O, which correspond to the inputs and outputs of a computation. To this abstract graph, we associate a physical resource state, the **graph state**, to be used in a computation: each node j of the graph corresponds to a single qumode and thus to a single pair $(\mathsf{Q}_j, \mathsf{P}_j)$.

For a given input state $|\psi\rangle$ on $|I|$ modes, the graph state associated to the open graph (G, I, O) can be constructed as follows:

1. Initialise each non-input qumode, $j \in I^{\mathrm{c}}$, in the squeezed state $|\eta\rangle$, resulting in a separable state of the form $|\eta\rangle^{\otimes|I^{\mathrm{c}}|} \otimes |\psi\rangle$.

2. For each edge in the graph between nodes j and k with weight $w(j,k) \in \mathbb{R}$, apply the entangling operation $\mathsf{CZ}_{j,k}\big(w(j,k)\big)$ between the corresponding qumodes.

See figure 1 for an example. Since our results will be dependant on the squeezing η used to construct the graph state but not on the input $|\psi\rangle$ (in that the unitary implemented by the MBQC is inpedendant of $|\psi\rangle$), we denote $|G(\eta)\rangle$ the resulting graph state.

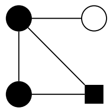
2 Results

2.1 CV-flow and MBQC procedure

Now, our aim, given an arbitrary open graph, is to find an order $\prec$ in which we can measure the non-output modes in such a way that we can always perform a correction like for gate teleporation (in a more general sense, we allow corrections to be applied on multiple nodes). In order to be able to implement more than a single computation, we further require that this order be independant both of the input state, and of the choice of measurements which are performed.

This leads to a technical definition but which is more general than the qubit case:

[2] An $\mathbb{R}$-edge-weighted graph is a pair (V, w) consisting of a set V of vertices (or nodes) and a function $w : V \times V \to \mathbb{R}$ which identifies the weight of each edge. Furthermore: if $w(j,k) = 0$ then there is no edge between j and k; for any $j, k \in V$, $w(j,k) = w(k,j)$ and $w(j,j) = 0$.



$$\underbrace{\mathsf{CZ}_{1,2}(a)\mathsf{CZ}_{1,3}(b)\mathsf{CZ}_{1,4}(c)\mathsf{CZ}_{3,4}(d)}_{\mathsf{G}}\left|\eta\right\rangle\left|\eta\right\rangle\left|\eta\right\rangle\left|\phi_4\right\rangle$$

Figure 1: Example of an open graph and the associated graph state. Black nodes are to be measured, white nodes are outputs, and the square node represents an input.

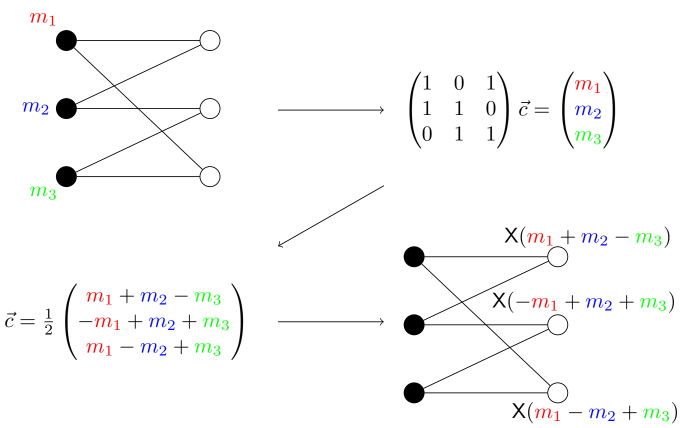


Figure 2: Example of a correction procedure based on a CV flow. We perform measurements on the black nodes of a graph state with measurement outcomes m_1, m_2, m_3, the white nodes are unmeasured (top left). The linear equations for the correction matrix of the graph (top right) is solvable for all measurement outcomes (bottom left), which gives a corresponding correction procedure (bottom right).

Definition 1 *Given an open graph (G, I, O) with a partial order $\prec$ over O^{c}, the* ***maximal correction subgraph*** *of node i is the directed bipartite graph $(M_i, \overline{C_i}, \rightsquigarrow)$ where:*

1. *$C_i \subseteq I^{\mathsf{c}}$ is the set of unmeasured non-input nodes $j \succ i$ connected to i. These are the nodes that can be used to correct a measurement on i.*

2. *$M_i \subseteq O^{\mathsf{c}}$ is the set of previously measured nodes $j \preceq i$ that are connected to C_i. These are the nodes on which corrections applied to C_i will have a back-action.*

3. *$\overline{C_i} \subseteq I^{\mathsf{c}}$ is the set of unmeasured non-input nodes $j \succ i$ connected to $\{i\} \cup M_i$. These additional nodes can be used to compensate for unwanted backactions to M_i.*

4. *We direct every edge in G between nodes in $(M_i, \overline{C_i}, \rightsquigarrow)$ in the direction $M_i \rightsquigarrow \overline{C_i}$.*

The ***correction matrix*** *A_i of node i is the $|M_i| \times \left|\overline{C_i}\right|$ bi-adjacency matrix corresponding to its maximal correction subgraph and given by $(A_i)_{jk} := w(j,k)$ (the weight of the edge from j to k).*

The correction matrix encodes what back-action a correction applied to the measured nodes will have on the previously measured nodes. It allows us to determine how to apply a correction on a specific node by controlling this back-action.

Definition 2 *An open graph (G, I, O) has a CV-flow if there exists a partial order $\prec$ on the nodes of G such that for every non-output node $i \in O^{\mathsf{c}}$ and all $m \in \mathbb{R}$ the linear equation*

$$A_i \vec{c}_i = \begin{pmatrix} m \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \textit{has solutions} \quad \vec{c}_i \in \mathbb{R}^n, \tag{10}$$

where A_i is the bi-adjacency matrix of the maximal correction subgraph of node i, and n the number of unmeasured nodes connected to i.

If a graph has a CV-flow, we can use the following flow measurement procedure to implement the MBQC:

1. Prepare the squeezed graph state $|G(\eta)\rangle$.

2. Successively measure the non-output nodes in the graph, in any order which is a linear extension of $\prec$, in the basis corresponding to $\mathsf{U}(\alpha, \beta, \gamma)$—for example, by applying $\mathsf{U}(\alpha, \beta, \gamma)$ and measuring P.

3. At each step, correct for the measurement result m_i by applying the correction

$$\mathsf{C}_j(m_j) := \mathsf{X}_{f(j)}(-m_j) \prod_{k \in N(f(j)) \setminus \{j\}} \mathsf{Z}_k(-m_j). \tag{11}$$

where we apply the translation operators on every as-of-yet unmeasured node.

See figure 2 for a simple example. We call $\mathsf{O}_\eta(\vec{\alpha}, \vec{\beta}, \vec{\gamma})$ the operator resulting from this finitely squeezed procedure.

Then our first result is that there is never a contradiction in running this MBQC procedure:

Theorem 3 *The CV-flow MBQC procedure is* runnable, *in that at no step does a correction depend on the outcome of measurements that have yet to be performed.*

2.2 Circuit extraction and convergence

Our second set of results concern the convergence of the MBQC procedure in the infinite squeezing limit. This convergence proof is obtained via an explicit circuit extraction scheme that is of independant interest for depth complexity analysis. Since circuit extraction methods are well known for open graphs with causal flow, our method for graphs with CV-flow uses a "peeling" technique: 1. we identify part of the graph with an equivalent graph

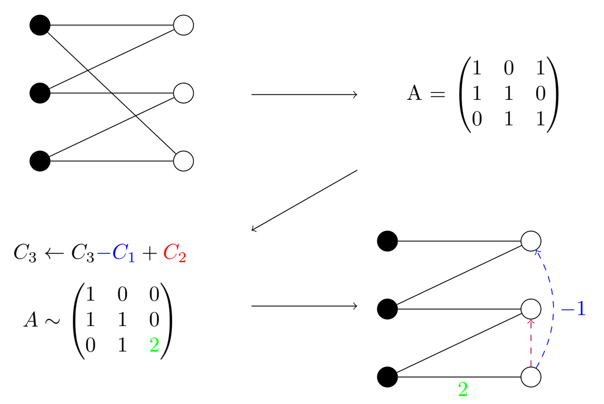


Figure 3: An example of the reduction of CV-flow to causal flow. Starting from a graph with CV flow (see figure 2), an upper triangularisation of the correction matrix gives a graph with causal flow (bottom right), up to additional weighted CX gates acting on the outputs (directed edges).

with causal flow; 2. we then extract this part of the circuit and remove the subgraph, repeating until the whole circuit has been extracted, as was done in [10] for qubits.

That this is possible comes down to the existence of decompositions of the open graph into subgraphs that respect the CV-flow order $\prec$:

Definition 4 *Let (G, I, O) be a graph with CV-flow $(c, \prec)$. A corresponding **layer decomposition** of (G, I, O) is a partition $\{V_k\}_{k=1}^N$ of O^c such that if $i \in V_m$, $j \in V_n$ and $i \prec j$ then $n < m$.*

Proposition 5 *If (G, I, O) is an open graph with CV-flow, and V is the first layer in a corresponding layer decomposition, then (G, I, O) is approximately equivalent to a graph with a causal flow from V to O, up to weighted CX gates acting in O, and reordering the nodes in V.*

This result is obtained by noting that linear operations on the column space of the correction matrix correspond to *controlled corrections* of the form

$$\mathsf{CX}_{j,k}(s) \prod_{\ell \in N(k)} \mathsf{CZ}_{j,l}(s). \tag{12}$$

The additional CZ operations can be interpreted as edges in a new open graph whose associated state is obtained from the original graph state by adding s to the weight of every edge $j \to \ell$, so that this operation can be viewed as a local complementation for $\mathbb{R}$-edge-weighted graphs. Then, there is an additional $\mathsf{CX}(s)$ gate in this new representation for the two open graphs to represent the same graph state. This gate is represented in the graphical as a directed edge, see figure 3.

The fact that the additional controlled gates act only on the outputs is crucial: it allows us reduce O_η to a sequence of single-gate teleportation operations. Since the CX gates never appear in between a measurement and the corresponding CZ gate for the teleportation, nor do they act on the auxiliary squeezed states before they are consumed in the teleportation, the projections can be brought forward and the squeezed inputs delayed to obtain a single gate teleportation circuit within the larger circuit of the MBQC procedure. We have:

Theorem 6 *Suppose the open graph (G, I, O) has a CV-flow, and the same number of inputs as outputs. Then, for any choice of measurements $\vec{\alpha}, \vec{\beta}, \vec{\gamma} \in \mathbb{R}^{O^c}$ the measurement procedure converges to a unitary U,*

$$\lim_{\eta \to +\infty} \mathsf{O}_\eta(\vec{\alpha}, \vec{\beta}, \vec{\gamma}) = \mathsf{U}(\vec{\alpha}, \vec{\beta}, \vec{\gamma}), \tag{13}$$

in the strong operator topology.

The circuit extraction scheme is strongly inspired by [10], and the resulting unitary U written in terms of (3). In other words, for a given input $|\phi\rangle \in \mathcal{S}(\mathbb{R})$ and choice of measurements $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$, it is possible to make the output $\mathsf{O}_\eta |\phi\rangle$ arbitrarily close to $\mathsf{U}|\phi\rangle$ by using a strong enough squeezing. In general, strong operator convergence is the best we can hope for in CV. However, when the total energy of the computation is bounded, this result can be strengthened to uniform convergence [22].

2.3 Comparing CV-flow and g-flow

For qubits, the notion of g-flow is the analogue of CV-flow. It gives rise to an MBQC procedure entirely analogous to the CV case, and is defined as:

Definition 7 *An open graph (G, I, O) has a **generalised flow**, or gflow, if there exists a map $g : I^c \to \mathcal{P}(O^c)$ and a partial order $\prec$ over G such that for all $i \in I^c$:*

- *if $j \in g(i)$ and $i \neq j$ then $i \prec j$;*
- *if $j \prec i$ then $j \notin \mathrm{Odd}(g(i))$.*

Even when both are well-defined (the open graph is unweighted or $\mathbb{Z}_2$-edge-weighted) it is not immediately clear if gflow and CV-flow are equivalent properties or not, or even if one is strictly stronger than the other. We construct counterexamples to either implication, showing that these are indeed entirely independant properties. Thus, a graph can have both (as in the case of flow), either or neither.

We have:

Proposition 8 *CV-flow and g-flow are not equivalent properties. A given open graph can have both, either or neither.*

For examples, see figure 4.

2.4 Generalised flow for qudits

We also note that our methods work for qudits whenever the local dimension d is prime (i.e. whenever $\mathbb{Z}_d$ is a field). Then, using the standard generalisation of the Pauli matrices to d-dimensions, essentially all the same commutation relations hold. As a consequence, we can perform gate teleportation using corrections weight by elements $m \in \mathbb{Z}_d$.

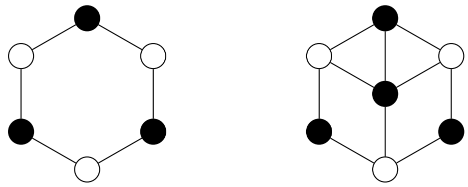


Figure 4: Examples of graphs states that have either CV-flow (left) or g-flow (right) but not both. There are no inputs–all nodes are prepared in an auxiliary state. The black nodes are to be measured and the white nodes are outputs which are left unmeasured at the end of the MBQC.

The correction matrices then have elements in $\mathbb{Z}_d$, and all the linear operations used for CV-flow work identically. In particular, since our circuit extraction scheme is purely linear-algebraic, it works immediately in this case, further generalising the results of [10].

Conclusion

We have defined a notion of flow for continuous variables and proved that it can be used to obtain a desired unitary, provided sufficient squeezing resources are available. We have also obtained a circuit extraction scheme, which might allow further comparison of depth and size complexities between circuit models and MBQC, as has already been obtained in the DV case. We have not considered the question of convergence rates in terms of the squeezing resources available nor the precision of measurements. These are highly dependant on the choice of measurements, auxiliary teleportation states, the topology of the graph, as well as the input state itself.

There are further extensions possible to the flow framework. In particular, one might consider Hilbert spaces over arbitrary locally compact Hausdorff fields and these come equipped with a different unitary group of translations. The cases considered in this article correspond to $\mathbb{R}$, $\mathbb{Z}_2$ and $\mathbb{Z}_d$. Then, one is interested in general in the case where the edges of the graph are weighted with elements of an arbitrary field $\mathbb{F}$, and the correction equations are solved in the $\mathbb{F}$-linear space $\mathbb{F}^n$.

References

[1] Robert Raussendorf and Hans J. Briegel. "A One-Way Quantum Computer". In: *Physical Review Letters* 86.22 (May 2001), pp. 5188–5191. DOI: 10.1103/PhysRevLett.86.5188.

[2] Robert Raussendorf and Hans Briegel. "Computational Model Underlying the One-Way Quantum Computer". In: *arXiv:quant-ph/0108067* (Aug. 2001).

[3] Vincent Danos and Elham Kashefi. "Determinism in the One-Way Model". In: *Physical Review A* 74.5 (Nov. 2006), p. 052310. DOI: 10.1103/PhysRevA.74.052310.

[4] Vincent Danos, Elham Kashefi, and Prakash Panangaden. "The Measurement Calculus". In: *Journal of the ACM* 54.2 (Apr. 2007), 8–es. DOI: 10.1145/1219092.1219096.

[5] D. E. Browne et al. "Generalized Flow and Determinism in Measurement-Based Quantum Computation". en. In: *New Journal of Physics* 9.8 (Aug. 2007), pp. 250–250. DOI: 10.1088/1367-2630/9/8/250.

[6] Anne Broadbent and Elham Kashefi. "Parallelizing Quantum Circuits". en. In: *Theoretical Computer Science* 410.26 (June 2009), pp. 2489–2510. DOI: 10.1016/j.tcs.2008.12.046.

[7] Joseph F. Fitzsimons and Elham Kashefi. "Unconditionally Verifiable Blind Quantum Computation". en. In: *Physical Review A* 96.1 (July 2017). DOI: 10.1103/PhysRevA.96.012303.

[8] Atul Mantri et al. "Flow Ambiguity: A Path Towards Classically Driven Blind Quantum Computation". en. In: *Physical Review X* 7.3 (July 2017). DOI: 10.1103/PhysRevX.7.031004.

[9] Damian Markham and Elham Kashefi. "Entanglement, Flow and Classical Simulatability in Measurement Based Quantum Computation". In: *arXiv:1311.3610 [quant-ph]* (Nov. 2013).

[10] Jisho Miyazaki, Michal Hajdušek, and Mio Murao. "An Analysis of the Trade-off between Spatial and Temporal Resources for Measurement-Based Quantum Computation". In: *Physical Review A* 91.5 (May 2015), p. 052302. DOI: 10.1103/PhysRevA.91.052302.

[11] Mehdi Mhalla et al. "Which Graph States Are Useful for Quantum Information Processing?" en. In: *Theory of Quantum Computation, Communication, and Cryptography*. Ed. by Dave Bacon, Miguel Martin-Delgado, and Martin Roetteler. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, 2014, pp. 174–187. DOI: 10.1007/978-3-642-54429-3_12.

[12] Ross Duncan and Simon Perdrix. "Rewriting Measurement-Based Quantum Computations with Generalised Flow". en. In: *Automata, Languages and Programming*. Ed. by David Hutchison et al. Vol. 6199. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 285–296. DOI: 10.1007/978-3-642-14162-1_24.

[13] Ross Duncan et al. "Graph-Theoretic Simplification of Quantum Circuits with the ZX-Calculus". en. In: *arXiv:1902.03178 [quant-ph]* (Feb. 2019).

[14] Niel de Beaudrap et al. "Pauli Fusion: A Computational Model to Realise Quantum Transformations from ZX Terms". en. In: *arXiv:1904.12817 [quant-ph]* (Apr. 2019).

[15] Seth Lloyd and Samuel L. Braunstein. "Quantum Computation over Continuous Variables". In: *Physical Review Letters* 82.8 (Feb. 1999), pp. 1784–1787. DOI: 10.1103/PhysRevLett.82.1784.

[16] Samuel L. Braunstein and Peter van Loock. "Quantum Information with Continuous Variables". In: *Reviews of Modern Physics* 77.2 (June 2005), pp. 513–577. DOI: 10.1103/RevModPhys.77.513.

[17] Jing Zhang and Samuel L. Braunstein. "Continuous-Variable Gaussian Analog of Cluster States". en. In: *Physical Review A* 73.3 (Mar. 2006). DOI: 10.1103/PhysRevA.73.032318.

[18] Nicolas C. Menicucci et al. "Universal Quantum Computation with Continuous-Variable Cluster States". In: *Physical Review Letters* 97.11 (Sept. 2006). DOI: 10.1103/PhysRevLett.97.110501.

[19] Daniel Gottesman, Alexei Kitaev, and John Preskill. "Encoding a Qubit in an Oscillator". In: *Physical Review A* 64.1 (June 2001). DOI: 10.1103/PhysRevA.64.012310.

[20] Tom Douce et al. "Probabilistic Fault-Tolerant Universal Quantum Computation and Sampling Problems in Continuous Variables". In: *Physical Review A* 99.1 (Jan. 2019), p. 012344. DOI: 10.1103/PhysRevA.99.012344.

[21] Yunong Shi, Christopher Chamberland, and Andrew W. Cross. "Fault-Tolerant Preparation of Approximate GKP States". en. In: (May 2019).

[22] Stefano Pirandola, Riccardo Laurenza, and Samuel L. Braunstein. "Teleportation Simulation of Bosonic Gaussian Channels: Strong and Uniform Convergence". In: *The European Physical Journal D* 72.9 (Sept. 2018), p. 162. DOI: 10.1140/epjd/e2018-90253-1.