Reachability Analysis of Infinite Markov Chain

Internship Proposal

Supervisor: Alain Finkel, Serge Haddad, Lina Ye

Start date: from Spring 2024
Duration: 5 to 6 months
Remuneration: to be discussed
Dominant: Computer science
Host lab: LMF - Laboratoire Méthodes Formelles
Université Paris-Saclay, CNRS, ENS Paris-Saclay
Contact: alain.finkel@lmf.cnrs.fr; serge.haddad@lmf.cnrs.fr;lina.ye@lmf.cnrs.fr

1 Background

1.1 Reachability Probability of Infinite Markov Chains

Recently, many works have been done to verify the infinite-state Markov chains obtained from probabilistic versions of automata extended with unbounded data (like stacks, channels, counters and clocks). In finite Markov chains, there is a well-known algorithm for computing exactly the reachability probabilities in polynomial time. In infinite Markov chains, one aims at computing the reachability probability up to an arbitrary precision (known as the CRP-problem). There are (at least) two possible research directions:

- The first one is to consider the Markov chains associated with a particular class of probabilistic models (like probabilitic pushdown automata or probabilistic Petri nets) and some specific target sets and to exploit the properties of these models to design a CRP-algorithm [2].
- The second one consists in exhibiting a property of Markov chains, such as *decisiveness* [1] or *divergence* introduced by us in [4], that yields a generic algorithm for solving the CRP problem and then looking for models that generate Markov chains that fulfill this property.

In most of the works, the probabilistic models associate a constant weight for transitions and get transition probabilities by normalizing these weights among the enabled transitions in the current state. This forbids to model phenomena like congestion in networks (resp. performance collapsing in distributed systems) when the number of messages (resp. processes) exceeds some threshold leading to an increasing probability of message arrivals (resp. process creations) before message departures (resp. process terminations).

To handle this, in our recent works [3, 4], we have studied classical probabilistic models associated with dynamic weights depending on the current state for both decisiveness and divergence properties.

1.2 Decisiveness and Divergence

Decisiveness property is introduced by [1] in order to approximately calculate the reachability probability of infinite Markov chains. Precisely, given a Markov chain, an initial state s_0 as well as a target subset of states A, this Markov chain is decisive with respect to s_0 and A if almost surely a random path ρ starting from s_0 will reach A or some state s' from which A is unreachable. When a Markov chain is decisive and its reachability problem is decidable, the algorithm presented in [3] can calculate approximately its reachability probability from s_0 to A.

We have established the equivalence relationship between decisiveness and recurrence properties, which allows to apply standard criteria for recurrence in order to check decisiveness. However, what if a given Markov chain is not recurrent? In order to cover this case, we introduce, in [4] a new property of Markov chains w.r.t. s_0 and A called divergence: given some precision θ, one can discard a set of states with either a small probability to be reached from s_0, identified by a function called f_0, or a small probability to reach A, recognised by another function called f_1, such that the remaining subset of states is finite and thus allows for an approximate computation of the reachability probability up to θ. For divergent Markov chains, we provide a generic algorithm for the CRP-problem that does not require the decidability of the reachability problem [4].

2 Research Direction

We have studied both decisiveness and divergence properties, separately, of some classical probabilistic models with dynamic weights (more details please see the long version of [3, 4]). For this internship, the candidate will further investigate one of the following directions:

- Establish the status of decidability of decisiveness and/or divergence for classical probabilistic models not yet studied, including both static and dynamic weights;
- Study the relationship between decisiveness and divergence and formalise it. Furthermore, characterize the class of models for which the two properties are exclusive or one implies the other;
- Investigate some heuristics to find out f_0 and/or f_1;
- Implement the CRP-algorithm for the divergent models (probabilistic open channel systems and/or increasing pushdown automata) exhibited in [4].

3 Candidate profile

The candidate needs to have background of computer science or probability, ideally with knowledge of Markov chains or formal methods for verification. However, the most important aspect to apply this internship is the motivation of the candidate, who wants really to get engaged in this topic.

References

1. Abdulla, P.A., Henda, N.B., Mayr, R.: Decisive markov chains. Log. Methods Comput. Sci. **3**(4) (2007). https://doi.org/10.2168/LMCS-3(4:7)2007
2. Brázdil, T., Esparza, J., Kiefer, S., Kucera, A.: Analyzing probabilistic pushdown automata. Formal Methods Syst. Des. **43**(2), 124–163 (2013). https://doi.org/10.1007/S10703-012-0166-0
3. Finkel, A., Haddad, S., Ye, L.: About decisiveness of dynamic probabilistic models. In: Pérez, G.A., Raskin, J. (eds.) 34th International Conference on Concurrency Theory, CONCUR 2023, September 18-23, 2023, Antwerp, Belgium. LIPIcs, vol. 279, pp. 14:1–14:17. Schloss Dagstuhl - Leibniz-Zentrum für Informatik (2023). https://doi.org/10.4230/LIPICS.CONCUR.2023.14, the long version is available on https://doi.org/10.48550/ARXIV.2305.19564
4. Finkel, A., Haddad, S., Ye, L.: Introducing divergence for infinite probabilistic models. In: Bournez, O., Formenti, E., Potapov, I. (eds.) Reachability Problems - 17th International Conference, RP 2023, Nice, France, October 11-13, 2023, Proceedings. Lecture Notes in Computer Science, vol. 14235, pp. 127–140. Springer (2023). https://doi.org/10.1007/978-3-031-45286-4_10, and the long version is available on https://doi.org/10.48550/ARXIV.2308.08842