

Algorithmics and complexity

TD 6/7 – Packing Problem

Training exercises

Note: The solution elements given here are not complete. Their purpose is only to guide you. We encourage you to write a proper answer, as you would do for the exam. If any question remains, feel free to ask your tutorial supervisor for help.

Exercise 1 : Limited Time

The company no longer wants to rent external servers: it therefore has only M servers. Each server is available B minutes a day but the goal of the company is now to reduce as much as possible the time of use of its servers. She wants to write a program that allows her to determine if it is possible to process the requests in a fixed time $T \leq B$.

Question 1

Give a formal definition of this new decision problem.

Solution elements :

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Inputs :

- a set O of N objects of sizes $o_i \in \mathbb{N}$
- $T \in \mathbb{N}$ the fixed size not to exceed
- M the number of available bags

Question : Is there an assignment of the N objects to the M bags, such that the sum of sizes of elements in each bag is less or equal to T .

The decision problem is the same except having T instead of B .

Question 2

What is the optimization problem that the company wants to solve?

Solution elements :

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Inputs :

- a set O of N objects of sizes $o_i \in \mathbb{N}$
- M a number of available bags

Question : What is the **smallest value** of T such that it exists an assignment $(f : [1, N] \rightarrow [1, M])$ with $\forall j \in \text{img}(f). \sum_{i \in f^{-1}(\{j\})} o_i \leq T$.

Question 3

Propose a greedy algorithm to solve this problem.

Solution elements :

We can, for instance, put the object in the bag which has systematically the largest remaining space (a kind of Worst-fit). **Write the corresponding Code in Python and test it.**